Q.P Code **D 122905**	Total Pages **3**	Name **603265**
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SECOND SEMESTER (CUFYUGP) DEGREE EXAMINATION, APRIL 2025

MATHEMATICS

MAT2MN105 Vector Spaces and Linear Transformations

2024 Admission Onwards

Maximum Time :2 Hours	**Maximum Marks :70**

Section A

All Question can be answered. Each Question carries 3 marks (Ceiling : 24 Marks)

1	Let V be a vector space, $\mathbf{u}$ a vector in V, and k a scalar ; then: prove that $\mathbf{0u} = \mathbf{0}$
2	If V is any vector space, and if $W = \{\mathbf{0}\}$ is the subset of V that consists of the zero vector only, then show that W is a subspace of V
3	Explain linearly independent and linearly dependent set of vectors. Give Examples
4	Show that the polynomials $1 + t, 2$ form a basis for $\mathbf{P_1}$.
5	Define dimension of a vector space. Find the dimension of all $n \times n$ diagonal matrices.
6	Find the image of $\mathbf{x} = (1, 1)$ under a rotation of $\pi/6$ radians about the origin.
7	Find the kernel of the linear transformation $T : R^2 \to R^2$ by $T(x_1, x_2) = (2x_1+x_2, 4x_1+2x_2)$
8	Define Eigen value and vector of a matrix. Write the eigen values of $n \times n$ diagonal matrix
9	Find the eigen values of the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$
10	Write a non diagonalizable 2×2 matrix with eigen value 2 only.

Section B

All Question can be answered. Each Question carries 6 marks (Ceiling : 36 Marks))

11	Show that the set M_{mn} of all $m \times n$ matrices with the usual operations of addition and scalar multiplication is a vector space over R.
12	Solve the system and then give a geometric description of the solution set. $$\begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \\ 3 & -6 & 9 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
13	Determine whether the vectors $(-3, 0, 4), (5, -1, 2), (1, 1, 3)$ are linearly independent or are linearly dependent in R^3.
14	Show that the following vectors do not form a basis for $\mathbf{P_2}$. $$1 - 3x + 2x^2,\ 1 + x + 4x^2,\ 1 - 7x$$
15	Use matrix multiplication to find the image of the vector (2, -1, 2) if it is rotated (a) $30°$ clockwise about the positive $x-$ axis. (b) $45°$ counterclockwise about the positive $y-$ axis.
16	Sketch the image of the unit square under multiplication by the invertible matrix $A = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}$. Label the vertices of the image with their coordinates, and number the edges of the unit square and their corresponding images.
17	Find the eigenvalues of $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{bmatrix}$
18	Show that the following matrix is not diagonalizable: $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ -3 & 5 & 2 \end{bmatrix}$

Section C

Answer any ONE. Each Question carries 10 marks (1x10=10 Marks))

19	Consider the vectors $\mathbf{u} = (1, 2, -1)$ and $\mathbf{v} = (6, 4, 2)$ in R^3. Show that $\mathbf{w} = (9, 2, 7)$ is a linear combination of $\mathbf{u}$ and $\mathbf{v}$ and that $\mathbf{w} = (4, -1, 8)$ is not a linear combination of $\mathbf{u}$ and $\mathbf{v}$.
20	Find a matrix P that diagonalizes $A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$