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Name..................................

Reg. No..................................

FIRST SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION NOVEMBER 2020

Mathematics

MAT 1C 01—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type Questions)

Answer **all** *questions (1 - 12).*
Each question carries 1 *mark.*

1. $\lim\limits_{x \to \infty} x \sin\left(\frac{1}{x}\right) = ...$

2. $\lim\limits_{x \to 0} \frac{\sin(2+x) - \sin 2}{x} = ...$

3. Define removable discontinuity.

4. State the condition(s) for local maximum of the function $y = f(x)$.

5. What is (are) the vertical asymptote(s) of the curve $xy^3 - 2xy^2 - 2y^3 - 4 = 0$.

6. State Rolle's theorem.

7 Find $\frac{d}{dx}(\cosh(3x-2))$.

8. State the second derivative test for concavity of a function $y = f(x)$.

9. State the mean value theorem for definite integral.

10. $\sum\limits_{k=1}^{4} \left(k^2 - 3k\right) = ...$

11. Let f be a continuous function on $[a,b]$. Then what is the average value of f on $[a,b]$.

12. Area bounded by the curves $y = f(x), y = g(x)$ and the ordinates $x = a$ and $x = b$ is given by ________.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions (13 - 24).*
Each question carries 2 *marks.*

13. If $\sqrt{3-2x} \le f(x) \le \sqrt{3-x}$, find $\lim_{x\to 0} f(x)$.

14. Find $\lim_{x\to 2} \dfrac{x-3}{x^2-4}$.

15. Find the equation of the tangent line to the curve $y=\sqrt{x}$ at $x = 4$.

16. Find the absolute extrema of $f(x)=x^{2/3}$ on $[-2, 3]$.

17. Find the points of inflection of the function $y = 2+\cos x, x \ge 0$.

18. Find $\lim_{x\to\infty} \dfrac{5x^2+8x-3}{3x^2+2}$.

19. Find the horizontal asymptotes of the graph of the function $f(x)=\dfrac{-8}{x^2-4}$.

20. Find the linearization of $f(x)=x^3-2x+3$ at $x = 2$.

21. Find $\dfrac{dy}{dx}$ if $y=\int_1^{x^2} \cos t\, dt$.

22. Find $\lim_{x\to 1} \dfrac{1-x}{\log x}$.

23. Find $\lim_{x\to\infty} x^{1/x}$.

24. Verify Rolle's theorem for the function $f(x)=\tan x$ in $[0,\pi]$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions (25 - 33).*
Each question carries 5 *marks.*

25. State and prove the rule for the limit of a sum.

26. Show that if a function f has a derivative at $x=c$, then show that f is continuous at $x=c$.

27. State and prove Rolle's theorem.

28. Verify mean value theorem for the function $f(x) = x^3 - 3x^2 + 2x$ in $\left[0, \frac{1}{2}\right]$.

29. Find the intervals on which $f(x) = -x^3 + 12x + 5, x \in [-3, 3]$ is increasing and decreasing.

30. Find all the asymptotes of $f(x) = \frac{x^2 - 3}{2x - 4}$.

31. Give an example of a function which is not Riemann integrable. Prove your claim.

32. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\pi/4$.

33. Verify the mean value theorem for integrals for the function $f(x) = \frac{x}{\sqrt{x^2 + 16}}$ in $[0, 3]$.

(6 × 5 = 30 marks)

Part D (Essay Questions)

Answer any **two** *questions (34 - 36).*
Each question carries 10 *marks.*

34. A dynamite blast blows a heavy rock straight up with a velocity of 160 ft/sec. It reaches a height of $s = 160t - 16t^2$ ft after t seconds.

    a) How high does the rock go ?

    b) What is the velocity and speed of the rock when it is at 256 ft above the ground on the way up? on the way down ?

    c) What is the acceleration of the rock at any time t during its flight ?

35. Sketch the graph of the function $y = x^4 - 4x^3 + 10$, by inspecting increasing, decreasing, concavity, points of inflection, local extrema etc.

36. a) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at 45° angle at the center of the cylinder. Find the volume of the wedge by slicing method.

    b) Find the area of the region bounded by the curves $y = x^2$ and $y = x^4 - 4x^2 + 4$.

(2 × 10 = 20 marks)