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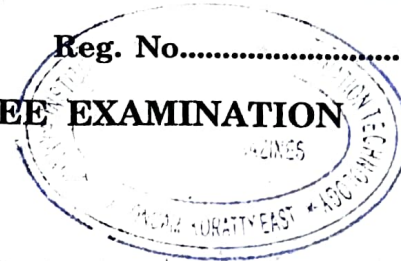
Name.....

Reg. No.....

FIRST SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2020

Mathematics

MAT 1C 01—MATHEMATICS



Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type Questions)

Answer all questions (1 - 12).
Each question carries 1 mark.

1. $\lim_{x \rightarrow \infty} x \sin\left(\frac{1}{x}\right) = \dots$
2. $\lim_{x \rightarrow 0} \frac{\sin(2+x) - \sin 2}{x} = \dots$
3. Define removable discontinuity.
4. State the condition(s) for local maximum of the function $y = f(x)$.
5. What is (are) the vertical asymptote(s) of the curve $xy^3 - 2xy^2 - 2y^3 - 4 = 0$.
6. State Rolle's theorem.
7. Find $\frac{d}{dx}(\cosh(3x-2))$.
8. State the second derivative test for concavity of a function $y = f(x)$.
9. State the mean value theorem for definite integral.
10. $\sum_{k=1}^4 (k^2 - 3k) = \dots$
11. Let f be a continuous function on $[a, b]$. Then what is the average value of f on $[a, b]$.
12. Area bounded by the curves $y = f(x), y = g(x)$ and the ordinates $x = a$ and $x = b$ is given by _____.

(12 × 1 = 12 marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** questions (13 - 24).

Each question carries 2 marks.

13. If $\sqrt{3-2x} \leq f(x) \leq \sqrt{3-x}$, find $\lim_{x \rightarrow 0} f(x)$.
14. Find $\lim_{x \rightarrow 2} \frac{x-3}{x^2-4}$.
15. Find the equation of the tangent line to the curve $y = \sqrt{x}$ at $x = 4$.
16. Find the absolute extrema of $f(x) = x^{2/3}$ on $[-2, 3]$.
17. Find the points of inflection of the function $y = 2 + \cos x, x \geq 0$.
18. Find $\lim_{x \rightarrow \infty} \frac{5x^2 + 8x - 3}{3x^2 + 2}$.
19. Find the horizontal asymptotes of the graph of the function $f(x) = \frac{-8}{x^2 - 4}$.
20. Find the linearization of $f(x) = x^3 - 2x + 3$ at $x = 2$.
21. Find $\frac{dy}{dx}$ if $y = \int_1^{x^2} \cos t \, dt$.
22. Find $\lim_{x \rightarrow 1} \frac{1-x}{\log x}$.
23. Find $\lim_{x \rightarrow \infty} x^{1/x}$.
24. Verify Rolle's theorem for the function $f(x) = \tan x$ in $[0, \pi]$.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** questions (25 - 33).

Each question carries 5 marks.

25. State and prove the rule for the limit of a sum.
26. Show that if a function f has a derivative at $x = c$, then show that f is continuous at $x = c$.
27. State and prove Rolle's theorem.

28. Verify mean value theorem for the function $f(x) = x^3 - 3x^2 + 2x$ in $\left[0, \frac{1}{2}\right]$.
29. Find the intervals on which $f(x) = -x^3 + 12x + 5, x \in [-3, 3]$ is increasing and decreasing.
30. Find all the asymptotes of $f(x) = \frac{x^2 - 3}{2x - 4}$.
31. Give an example of a function which is not Riemann integrable. Prove your claim.
32. Find the area between $y = \sec^2 x$ and $y = \sin x$ from 0 to $\pi/4$.
33. Verify the mean value theorem for integrals for the function $f(x) = \frac{x}{\sqrt{x^2 + 16}}$ in $[0, 3]$.
(6 × 5 = 30 marks)

Part D (Essay Questions)

Answer any two questions (34 - 36).

Each question carries 10 marks.

34. A dynamite blast blows a heavy rock straight up with a velocity of 160 ft/sec. It reaches a height of $s = 160t - 16t^2$ ft after t seconds.
- How high does the rock go ?
 - What is the velocity and speed of the rock when it is at 256 ft above the ground on the way up? on the way down ?
 - What is the acceleration of the rock at any time t during its flight ?
35. Sketch the graph of the function $y = x^4 - 4x^3 + 10$, by inspecting increasing, decreasing, concavity, points of inflection, local extrema etc.
36. a) A curved wedge is cut from a cylinder of radius 3 by two planes. One plane is perpendicular to the axis of the cylinder. The second plane crosses the first plane at 45° angle at the center of the cylinder. Find the volume of the wedge by slicing method.
- b) Find the area of the region bounded by the curves $y = x^2$ and $y = x^4 - 4x^2 + 4$.

(2 × 10 = 20 marks)