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Name................................

Reg. No................................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2021

Mathematics

MTS 1C 01—MATHEMATICS—I

(2021 Admissions)

Time : Two Hours

Maximum : 60 Marks

Section A

*Answer at least **eight** questions.*

Each question carries 3 marks.

All questions can be attended.

Overall Ceiling 24.

1. Calculate the slope of the tangent line to the graph of $f(x) = x^2 + 1$ when $x = -1$.

2. Find $\lim\limits_{x \to 1} \dfrac{x^2 + x - 2}{x^2 - x}$.

3. Find the derivative of $y = \sqrt{x}$ for $x > 0$.

4. Find $\dfrac{d}{dx}\left[\cos\left(\sqrt{1 + \cos x}\right)\right]$.

5. Find the linearization of $f(x) = \cos x$ at $x = \pi/2$.

6. Show that there is a number c such that $c^3 - c^2 = 10$.

7. Find $\lim\limits_{t \to 0} \cos\left(\dfrac{x}{\sqrt{19 - 3\sec 2t}}\right)$.

8. Suppose that f is differentiable on the whole real line and that $f'(x)$ is constant. Prove that f is linear.

Turn over

9. Find the critical points of $f(x) = 3x^4 - 8x^3 + 6x^2 - 1$.

10. Find the inflection points of $f(x) = x^2 + (1/x)$.

11. Using limits of Riemann sums, establish the equation $\int_a^b c\, dx = c(b-a)$, where c is a constant.

12. Find $\int_0^2 \left(\dfrac{t^2}{4} - 7t + 5\right) dt$.

$(8 \times 3 = 24 \text{ marks})$

Section B

*Answer at least **five** questions.*
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.

13. Find $\displaystyle\lim_{h \to 0} \dfrac{\sqrt{2+h} - \sqrt{2}}{h}$.

14. Show that the line $y = mx + b$ is its own tangent at any point $(x, mx + b)$ on the line.

15. Assume that oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 1 ft/s. How fast is the area of the spill increasing when the radius of the spill is 20 ft?

16. Use implicit differentiation to find d^2y/dx^2 if $5x^3 - 7y^2 = 10$.

17. Find the maximum and minimum points and values for the function $f(x) = \left(x^2 - 8x + 12\right)^4$ on the interval $[-10, 10]$.

18. Use l'Hôpital's Rule to find $\displaystyle\lim_{x \to 0} \dfrac{\sin x - x}{x^3}$.

19. Find the area of the region between the x-axis and the graph of $f(x) = x^3 - x^2 - 2x$, $-1 \le x \le 2$.

$(5 \times 5 = 25 \text{ marks})$

Section C

*Answer any **one** question.*
The question carries 11 marks.

20. (a) Find the area of the region in the first quadrant that is bounded above by $y = \sqrt{x}$ and below by the x-axis and the line $y = x - 2$.

 (b) Find $\dfrac{dy}{dx}$ if $y = \int_1^{x^2} \cos t\, dt$.

21. (a) Find the absolute extrema of $h(x) = x^{2/3}$ on $[-2, 3]$.

 (b) Find the volume of the solid generated by the revolution about the x-axis of the loop of the curve $y^2 = x^2 \dfrac{3a - x}{a + x}$.

 (c) Evaluate $\displaystyle\lim_{x \to 0} \left(\dfrac{1}{x^2} - \dfrac{1}{\sin^2 x}\right)$.

$(1 \times 11 = 11 \text{ marks})$