D 32374 **(Pages : 3)** **Name..................................**

Reg. No...............................

FIRST SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MTS 1C 01—MATHEMATICS—I

(2019—2022 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 20 marks.

1. Calculate the slope of the tangent line to the graph of $y = x^2$ at $x = 1$.

2. Find $\lim_{x \to -5} \frac{x^2 + 3x - 10}{x + 5}$.

3. If f has a derivative at $x = c$, then prove that f is continuous at $x = c$.

4. Find the derivative of $y = \frac{2x + 5}{3x - 2}$.

5. Find the linearization of $f(x) = x^4$ when $x = 1$.

6. Find $\frac{d}{dx}\left[\tan\left(x^2 + 1\right)\right]$.

7. Find $\lim_{x \to 0} \frac{(1 + x)^n - 1}{x}$.

8. Find points of inflection on the curve $y = 3x^4 - 4x^3 + 1$.

9. Find the intervals on which the function $g(t) = -t^2 - 3t + 3$ is increasing and decreasing.

Turn over

10. Evaluate $\sum_{k=1}^{7} - 2k$.

11. Using limits of Riemann sums, establish the equation $\int_a^b c\, dx = c\,(b - a),$ where c is a constant.

12. Find $\int_1^2 \frac{x^2 + 2x + 2}{x^4}\, dx.$

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 30 marks.

13. If $\lim_{x \to 4} \frac{f(x) - 5}{x - 2} = 1,$ find $\lim_{x \to 4} f(x).$

14. Show that the line $y = mx + b$ is its own tangent at any point $(x, mx + b)$ on the line.

15. An oil slick has area $y = 30x^3 + 100x$ square meters x minutes after a tanker explosion. Find the average rate of change in area with respect to time during the period from $x = 2$ to $x = 3$ and from $x = 2$ to $x = 2.1$. What is the instantaneous rate of change of area with respect to time at $x = 2$?

16. State and prove power rule for positive integers.

17. Find the maximum and minimum points and values for the function $f(x) = \left(x^2 - 8x + 12\right)^4$ on the interval [– 10, 10].

18. Evaluate $\lim_{x \to 0} \left(\frac{1}{\sin x} - \frac{1}{x}\right).$

19. Find the area of the region in the first quadrant bounded by the line $y = x$.the line $x = 2$,the curve $y = 1/x^2$, and the axis.

Section C

Answer any **one** *question.*
Each question carries 10 *marks.*
Maximum 10 *marks.*

20. (a) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

(b) Evaluate $\frac{d}{dx}\int_0^{\sqrt{x}} \cos t \, dt$.

21. (a) Find the absolute maximum and minimum values of $f(x) = x^2$ on $[-2, 1]$.

(b) Evaluate $\lim_{x \to \frac{\pi}{2}} (\sin x)^{\tan x}$.

(c) State and prove the product rule of differentiation.