

C 4388-B

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Name.....

Reg. No.....

## SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

MAT 2C 02—MATHEMATICS—2

(2020 Admissions)

Time : Two Hours

Maximum : 60 Marks

## Section A

*Answer at least eight questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 24.*

1. Prove that  $\cosh^2 x - \sinh^2 x = 1$ .
2. Find the Cartesian form of the polar equation  $r = \frac{8}{1 - 2\cos\theta}$ .
3. Find the slope of the line tangent to the graph of  $r = 3\cos^2 2\theta$  at  $\theta = \pi/6$ .
4. Evaluate  $\int \sinh^2 x dx$ .
5. Show that  $\lim_{n \rightarrow \infty} \frac{2n}{n^2 + 1} = 0$ .
6. Test the convergence of the series  $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{4}} - \frac{1}{\sqrt{8}} + \frac{1}{\sqrt{16}} \dots$
7. Compute  $\|\cos x\|$  in  $C[0, 2\pi]$ .
8. Examine whether the set of vectors  $u_1 = \langle 1, 2, 3 \rangle$ ,  $u_2 = \langle 2, 4, 3 \rangle$ , and  $u_3 = \langle 3, 2, 1 \rangle$  is linearly independent or not.
9. Find the eigenvalues of the matrix  $A = \begin{bmatrix} 3 & 4 \\ -1 & 7 \end{bmatrix}$ .
10. Find the determinant of the matrix  $C = \begin{bmatrix} -1 & 2 & 9 \\ 2 & -4 & -18 \\ 5 & 7 & 27 \end{bmatrix}$ .

Turn over

11. Show that  $A = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$  is an orthogonal matrix.

12. Find the eigen values of the matrix  $A = \begin{bmatrix} 10 & 3 \\ 4 & 6 \end{bmatrix}$ .

(8 × 3 = 24 marks)

**Section B***Answer at least five questions.**Each question carries 5 marks.**All questions can be attended.**Overall Ceiling 25.*

13. Find the length of the curve  $y = \frac{4\sqrt{2}}{3}x^{3/2} - 1, 0 \leq x \leq 1$ .

14. Find the equation of the tangent line when  $t = 1$  for the curve  $x = t^4 + 2\sqrt{t}, y = \sin(t\pi)$ .

15. Find the length of the perimeter of the cardioid  $r = a(1 - \cos\theta)$ .

16. Use the Trapezoidal rule with  $n = 4$  to estimate  $\int_1^2 x^2 dx$ . Compare the estimate with the exact value of the integral.

17. Using Maclaurin's series expand  $\tan^{-1}x$ . Hence deduce the Gregory series  $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$

18. Show that the set  $B = \{(1, 2, 1), (2, 1, 0), (1, -1, 2)\}$  is a basis for  $\mathbb{R}^3$ .

19. Find the inverse of the matrix  $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 2 & 0 \\ 3 & -1 & 4 \end{bmatrix}$ .

(5 × 5 = 25 marks)

**Section C***Answer any one question.**The question carries 11 marks.*

20. (a) Evaluate  $\int_1^{\infty} \frac{\ln x}{x^2} dx$ , if it exists.

(b) Find the area of the region shared by the cardioids  $r = 2(1 + \cos\theta)$  and  $r = 2(1 - \cos\theta)$ .

21. (a) Solve :

$$x_1 + x_2 + x_3 + x_4 = 0$$

$$x_1 + 3x_2 + 2x_3 + 4x_4 = 0$$

$$2x_1 + x_3 - x_4 = 0.$$

(b) Find the eigen values of the matrix  $A = \begin{bmatrix} 1 & -6 \\ 2 & 2 \end{bmatrix}$ .

(1 × 11 = 11 marks)