C 43191 **(Pages : 3)** **Name.......................................**

Reg. No..................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2023

Mathematics

MTS 2C 02—MATHEMATICS—2

(2020—2022 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum marks that can be earned from this Section is 20.

1. Sketch the set of points whose polar co-ordinates (r, θ) satisfy the conditions $0 < r < 4$ and $-\pi/2 < \theta < \pi/2$.

2. Let $f(x) = x^2 + 2x + 3$. Restrict f to a suitable interval so that it has an inverse. Find the inverse $f^{-1}(x)$ of the given function.

3. Find the slope of the line tangent to the graph of $r = \cos(3\theta)$ at $\theta = \pi/3$.

4. Prove $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$.

5. Show that determinant of a square matrix A is the product of its eigenvalues

6. Find $\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$ using the notion of improper integrals.

7. Evaluate $\lim_{n \to \infty} \frac{n^2 + 1}{3n^2 + n}$.

8. Define rank of a matrix.

Turn over

9. Find A^2 using Cayley-Hamilton theorem, if $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$.

10. When will you say that a series $\sum_{n=1}^{\infty} a_n$ an converges to the sum s.

11. Use Doolittle's method to find an LU-factorization of $B = \begin{bmatrix} 1 & 1 & 1 \\ 3 & 1 & 2 \\ 1 & -1 & 1 \end{bmatrix}$.

12. Selecting a proper test of convergence, decide whether the series $\sum_{n=1}^{\infty} \frac{1}{n^2 - \ln n}$ converges or diverges.

Section B

Answer any number questions from this section.
Each question carries 5 marks.
Maximum that can be earned from this Section is 30.

13. Find the maxima and minima of $f(\theta) = 1 + 2\cos(\theta)$. Sketch the graph of $r = 1 + 2\cos\theta$ in the xy–plane.

14. Find :

    (a) $\frac{d}{dx}\left(\sinh^{-1}(3x)\right)$; and

    (b) $\int \sinh^{-1}(3x)\, dx$.

15. Prove that the length of the parabola $y = x^2$ from $x = 0$ to $x = 1$ is $\frac{1}{2}\left[\sqrt{5} + \frac{1}{2}\ln\left(2 + \sqrt{5}\right)\right]$.

16. A bouncing ball loses half of its energy on each bounce. The height reached on each bounce is proportional to the energy. Suppose that the ball is dropped vertically from a height of one meter. How far does it travel ?

17. Find the Taylor's series expansion for $\ln x$ around $x = 1$.

18. Find the rank of the matrix $A + 3I$ where $A = \begin{bmatrix} 0 & 1 & 1 & 7 \\ 1 & 0 & 1 & -3 \\ 4 & 1 & 0 & 3 \\ 4 & 1 & 0 & 3 \end{bmatrix}$.

19. The set $B = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$, where $\mathbf{u}_1 = \langle 1, 1, 1 \rangle$, $\mathbf{u}_2 = \langle 1, 2, 2 \rangle$, $\mathbf{u}_3 = \langle 1, 1, 0 \rangle$ is a basis for $\mathbb{R}^3$. Transform B into an orthonormal basis.

Section C

Answer **one** *question from this section.*
The question carries 10 *marks.*
Maximum that can be earned from this Section is 10.

20. (a) Find the surface area of a sphere of radius r using the method of integration.

(b) Use Simpson's rule of integration to evaluate $\int_0^1 \frac{dx}{x^2 + 1}$ and hence find an approximate value for π.

21. (a) If consistent solve the system using Gauss-Jordan elimination :

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 3 \\ 3x_1 + x_2 + x_3 + x_4 &= 4 \\ x_1 + 2x_2 + 2x_3 + 3x_4 &= 3 \\ 4x_1 + 5x_2 - 2x_3 + x_4 &= 16. \end{aligned}$$

(b) Diagonalize the matrix $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

(1 × 10 = 10 marks)