

C 81806

(Pages : 3)

Name.....

Reg. No.....

SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2020

Mathematics

MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer all twelve questions.

Each question carries 1 mark.

1. Find a formula for n th term of the sequence 1, -1, 1, -1,.....
2. Define a non-decreasing sequence.
3. Write $\tanh x$ in terms of exponential function.
4. State Sandwich theorem for sequences.
5. Find the domain of the function $w = \sqrt{y-x^2}$.
6. Define contour line of a function $f(x,y)$.
7. Find $\lim_{(x,y) \rightarrow (0,1)} \frac{2-xy+3}{x^2y+5xy-y^3}$.
8. Define absolute convergence of a series.
9. $\frac{d}{dx} \sinh 2x =$ _____.
10. Define level curve of a function.
11. Find $\lim_{n \rightarrow \infty} n^{1/n}$.
12. $\int_0^1 \sinh x dx =$ _____.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any nine questions.

Each question carries 2 marks.

13. Is the area under the curve $y = 1/\sqrt{x}$ from $x = 0$ to $x = 1$ finite? If so, what is it?
14. Evaluate $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$.

Turn over

15. Show that $\lim_{n \rightarrow \infty} \frac{1}{n} = 0$.
16. Determine whether the sequence $a_n = \frac{2^n 3^n}{n!}$ is non-decreasing and bounded from above.
17. Show that $f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0); \\ 0, & (x, y) = (0, 0); \end{cases}$ is continuous at every point except the origin.
18. Find the values of $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ at the point $(4, -5)$ if $f(x, y) = x^2 + 3xy + y - 1$.
19. Find the volume of the solid generated by revolving the region between y -axis and the curve $x = 2/y, 1 \leq y \leq 4$ about the y -axis.
20. Find $\int_0^{\ln 2} 4e^x \sin x dx$.
21. Graph the sets of points whose polar co-ordinates satisfies the conditions $1 \leq r \leq 2$ and $0 \leq \theta \leq \pi/2$.
22. Replace the polar equation $r = \frac{4}{2\cos\theta - \sin\theta}$ equivalent cartesian equation.
23. Find the spherical co-ordinates equation for the cone $z = \sqrt{x^2 + y^2}$.
24. Find $\frac{dw}{dt}$ if $w = xy + z, x = \cos t, y = \sin t, z = t$. What is the derivative's value at $t = 0$?

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any six questions.
Each question carries 5 marks.

25. Investigate the convergence of $\int_0^3 \frac{dx}{(x-1)^{2/3}}$.
26. Show that $(-1)^n \frac{n-1}{n}$ diverges.
27. Find the area of the region in the plane enclosed by the cardioid $r = 2(1 + \cos\theta)$.
28. Show that the p -series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if $p > 1$ and diverges if $p \leq 1$.

29. Find the linearization of $f(x, y, z) = x^2 - xy + 3\sin z$ at the point (x_0, y_0, z_0) .
30. Find $\frac{dw}{dt}$ if $w = xy + z, x = \cos t, y = \sin t, z = t$. What is the derivative's value at $t = 0$?
31. Find the area of the region that lies inside the circle $r = 1$ and outside the cardioid $r = 1 - \cos \theta$.
32. Show that $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$ if $f(x, y, z) = (x^2 + y^2 + z^2)^{-1/2}$.
33. Find the radius and interval of convergence of the series $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$.

(6 × 5 = 30 marks)

Part D (Essay Type)

*Answer any two questions.
Each question carries 10 marks.*

34. Evaluate $\int_2^{\infty} \frac{x+3}{(x-1)(x^2+1)} dx$.
35. Find the length of the curve $y = (x/2)^{2/3}$ from $x = 0$ to $x = 2$.
36. Find the lateral surface area of the cone generated by revolving the line segment $y = x/2, 0 \leq x \leq 4$ about the x -axis.

(2 × 10 = 20 marks)

