D 103771 **(Pages : 3)** Name.......................................

Reg. No...................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION
APRIL 2024

Mathematics

MAT 2C 02—MATHEMATICS—2

(2020—2023 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 20.

1. Find the cartesian co-ordinates of $(r, \theta) = (6, -\pi/8)$.

2. Let $y = x^3 + 2$. Find $\frac{dx}{dy}$ when $y = 3$.

3. Compute $\int \coth x \, dx$.

4. Find $\lim_{n \to \infty} \left(\frac{n^2 + 1}{3n^2 + n} \right)$.

5. Sum the series $\sum_{i=0}^{\infty} \frac{3^i - 2^i}{6^i}$.

6. Show that $\sum_{i=1}^{\infty} \frac{2}{4 + i}$ diverges.

Turn over

7. Verify that the basis $B = \left\{ \left\langle \frac{12}{13}, \frac{5}{13} \right\rangle, \left\langle \frac{5}{13}, \frac{-12}{13} \right\rangle \right\}$ is an orthonormal basis for $\mathbb{R}^2$.

8. Find the rank of $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 4 \\ 1 & 4 & 1 \end{bmatrix}$.

9. Evaluate determinant of $A = \begin{bmatrix} 2 & 4 & 7 \\ 6 & 0 & 3 \\ 1 & 5 & 3 \end{bmatrix}$.

10. Find the value of x such that the matrix $A = \begin{bmatrix} 4 & -3 \\ x & -4 \end{bmatrix}$ is its own inverse.

11. Find the eigenvalues of $A = \begin{bmatrix} 1 & 2 & 1 \\ 6 & -1 & 0 \\ -1 & -2 & -1 \end{bmatrix}$.

12. Verify that the matrix $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 0 & 1 & 1 \end{bmatrix}$ satisfies its characteristic equation.

Section B

Answer any number of questions.

Each question carries 5 *marks.*

Ceiling is 30.

13. Find the length of the graph of $f(x) = (x-1)^{3/2} + 2$ on $[1, 2]$.

14. Find the area of the surface obtained by revolving the graph of x^3 on $[0, 1]$ about the x-axis.

15. Show that the improper integral $\int_0^{\infty} \frac{e^{-x}}{\sqrt{x}}\, dx$ is convergent.

16. Let $f(x) = \cos x$. Evaluate $\int_0^{\pi/2} \cos x \, dx$ by the Simpson's rule, taking 10 equally spaced points.

17. Let $u_1 = \langle 1, -1, 1, -1 \rangle, u_2 = \langle 1, 3, 0, -1 \rangle$ be the vectors span a subspace W of $\mathbb{R}^4$. Use the Gram-Schmidt orthogonalization process to construct a orthonormal basis for the subspace W.

18. Find nontrivial solution for the homogeneous system of equations

$$\begin{aligned} 2x_1 - 4x_2 + 3x_3 &= 0 \\ x_1 + x_2 - 2x_3 &= 0. \end{aligned}$$

19. Find the inverse of $A = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{bmatrix}$.

Section C

Answer any **one** *questions.*

The question carries 10 *marks.*

20. Use Gaussian elimination or Gauss-Jordan elimination to solve

$$\begin{aligned} 2x_1 + x_2 + x_3 &= 3 \\ 3x_1 + x_2 + x_3 + x_4 &= 4 \\ x_1 + 2x_2 + 2x_3 + 3x_4 &= 3 \\ 4x_1 + 5x_2 - 2x_3 + x_4 &= 16. \end{aligned}$$

21. Determine whether the matrix $A = \begin{bmatrix} 9 & 1 & 1 \\ 1 & 9 & 1 \\ 1 & 1 & 9 \end{bmatrix}$ is diagonalizable. If so, find the matrix P that diagonalizes A and the diagonal matrix D such that $D = P^T AP$.

(1 × 10 = 10 marks)