D 31819 **(Pages : 3)** **Name.....**

Reg. No.....

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2022

Mathematics

MTS 3C 03—MATHEMATICS – 3

(2019 Admission Onwards)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Maximum 20 *marks.*

1. Find the derivative of the vector function $\vec{r}(t) = \sin t\,\hat{i} - e^{-t}\hat{j} + \left(3t^3 - 4\right)\hat{k}$.

2. If $z = 4x^3y^2 - 6x^2 + y^2 + 5$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

3. If $f(x,y) = e^{xy}$, find $\nabla f(x,y)$.

4. Find the level curve of $f(x,y) = y^2 - x^2$ passing through the point (–1, 2).

5. Find div $\vec{F}$ for $\vec{F} = \left(x^2y^3 - z^4\right)\hat{i} + 4x^5y^2z\hat{j} - y^4z^6\hat{k}$.

6. Evaluate $\int_1^3 \int_{-1}^1 (2x-4)\,dx$.

7. State Stoke's theorem.

8. Find the Jacobian of $x = r\cos\theta,\ y = r\sin\theta$.

9. Express $1 + i$ in polar form.

10. Evaluate $\lim_{z \to i} \frac{z^4 - 1}{z - i}$.

Turn over

11. Evaluate $\oint_C \frac{e^z}{z-3}\,dz$ where C is $|z|=1$.

12. Evaluate $\oint_C \bar{z}\,dz$ where C is $x = t, y = t^2, 0 \le t \le 1$.

Section B

Answer any number of questions.
Maximum 30 *marks.*

13. Use chain rule to find $\frac{\partial z}{\partial u}$ at $(\pi,1)$ for $z = x^2 - y^2 \tan x$, where $x = \frac{u}{v}, y = uv$.

14. Find an equation of the tangent plane to the graph of $z=\frac{x^2}{2}+\frac{y^2}{2}+4$ at (1, –1, 5).

15. Show that $\int_C \left(y^2 - 6xy + 6\right)dx + \left(2xy - 3x^2\right)dy$ is independent of any path C between (–1, 0) and (3, 4). Hence evaluate $\int_{(-1,0)}^{(3,4)} \left(y^2 - 6xy + 6\right)dx + \left(2xy - 3x^2\right)dy$.

16. Change the order of integration and hence evaluate $\int_0^4 \int_y^4 \frac{x}{x^2+y^2}\,dx\,dy$.

17. Show that $u(x,y) = x^3 - 3xy^2 - 5y$ is harmonic. Find the harmonic conjugate of u.

18. Evaluate $\int_C z^2 dz$ where C is the line $x = 2y$ from $z = 0$ to $z = 2 + i$.

19. Evaluate $\oint_C \frac{z+1}{z^4+4z^3}\,dz$ where C is $|z| = 1$.

Section C

Answer any **one** *question.*

Maximum 10 *marks.*

20. Use Green's theorem to evaluate $\oint_C \left(x^5 + 3y\right) dx + \left(2x - e^{y^3}\right) dy$ where C is the circle $(x-1)^2 + (y-5)^2 = 4$.

21. Find the volume bounded by the cylinder $x^2 + y^2 = 4$, the plane $y + z = 3$ and $z = 0$.

(1 × 10 = 10 marks)