D 92960 (Pages : 3) Name.....

Reg. No.....

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2020

Mathematics

MTS 3C 03—MATHEMATICS – 3

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. If $\bar{r}(t) = 2\cos t\,\hat{i} + 6\sin t\,\hat{j}$, find $\frac{d\bar{r}}{dt}$ at $t = \frac{\pi}{2}$.

2. Find the curvature of a circle whose radius is 2.

3. If $z = e^x \sin(xy)$, find $\frac{\partial^2 z}{\partial y^2}$.

4. Find the gradient of $f(x, y, z) = xy^2 + 3x^2 - z^3$ at (1, 1, 1).

5. Show that div $\bar{r} = 3$.

6. Evaluate $\int_2^4 \int_1^3 (40 - 2xy)\,dx\,dy$.

7. Use double integrals to find the area of the plane region enclose by the curves $y \sin x$ and $y = \cos x$ for $0 \le x \le \pi/4$.

8. Find the Jacobian of $u = \frac{y}{x^2}, v = xy$.

9. Sketch the graph of the region $|z - 2i| = 2$.

10. Write the real and imaginary part of $f(z) = \sin z$.

Turn over

11. Evaluate $\oint_C \frac{z^2}{z-1}\,dz$, where C is $|z| = 2$.

12. Evaluate $\int_C z\,dz$ where C is given by $x = t^2, y = t$ from $0 \le t \le 1$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Find the directional derivative of $F(x,y,z) = xy^2 - 4x^2y + z^2$ at $(1,-1,2)$ in the direction of $6\hat{i} + 2\hat{j} + 3\hat{k}$.

14. Find an equation of the tangent plane to the graph of $x^2 - 4y^2 + z^2 = 16$ at (2, 1, 4).

15. Use Green's theorem to evaluate $\oint_C (x^2 - y^2)\,dx + (2y - x)\,dy$, where C consists of the boundary of the region in the first quadrant that is bounded by $y = x^2$ and $y = x^3$.

16. Change the order of integration and hence evaluate $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\,dy\,dx$.

17. Use divergence theorem to evaluate $\iint_S (\vec{F} \cdot \hat{n})\,dS$ where $\vec{F} = xy\,\hat{i} + y^2\,z\hat{j} + z^3\hat{k}$ and S is the unit cube defined by $0 \le x \le 1, 0 \le y \le 1, 0 \le z \le 1$.

18. Evaluate $\oint_C \left(z + \frac{1}{z}\right) dz$, where C is the unit circle $|z| = 1$.

19. Evaluate $\oint_C \frac{z^4 - 3z^2 + 6}{(z+i)^3}\,dz$, where C is $|z| = 2$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Use Stoke's theorem to evaluate $\oint_C z\,dx + x\,dy + y\,dz$, where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$ counter clockwise as viewed from above.

21. Find the volume of the solid in the first octant bounded by the graphs of $z = 1 - y^2$, $y = 2x$ and $x = 3$.

(1 × 11 = 11 marks)