D 91751 (Pages : 4) Name.................................

Reg. No.................................

THIRD SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2020

Mathematics

MAT 3C 03—MATHEMATICS

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer all the **twelve** *questions.*
Each question carries 1 *mark.*

1. What do you mean by a homogeneous equation ?

2. Consider a system of linear equations in n unknowns with augmented matrix $M=[A, B]$. Then, the solution is unique if and only if rank (A).

3. What is the order of the differential equation $y\left(\frac{dy}{dx}\right)^2+8x=0$.

4. State Cayley-Hamilton theorem.

5. What is the determinant of a 2×2 matrix whose rank is 1 ?

6. What is the normal form of the matrix $\begin{pmatrix} 2 & 3 & 1 \\ 4 & 5 & 1 \end{pmatrix}$?

7. Define eigen value of a matrix.

8. Define divergence of a vector field.

9. Define gradient of a function.

10. Define the derivative of a vector function.

11. Define a smooth curve.

12. State Gauss's divergence theorem.

$(12 \times 1 = 12$ marks)

Turn over

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 marks.

13. Solve the initial value problem $y' = 3y, y(0) = 5.7$.

14. Find an integrating factor for $2\cosh x \cos y dx = \sinh x \sin y dy$ and solve it.

15. Find the angles of the triangle with vertices $(0,0,0), (1,2,3), (4,-1,3)$.

16. Find x, y, z, t where $3\begin{pmatrix} x & y \\ z & t \end{pmatrix} = \begin{pmatrix} x & 6 \\ -1 & 2t \end{pmatrix} + \begin{pmatrix} 4 & x+y \\ z+t & 3 \end{pmatrix}$.

17. Solve the system :

$$x - 3y = 4$$
$$-2x + 6y = -8.$$

18. Show that circle of radius a has curvature $\frac{1}{a}$.

19. Find a unit normal vector n of the cone of revolution $z^2 = 4\left(x^2 + y^2\right)$ at the point $(1, 0, 2)$.

20. Find the directional derivative of $f = x^2 + y^2 - z$ at $(1, 1, -2)$ in the direction of $(1, 1, 2)$.

21. Show that curl $(u + v) =$ curl u + curl v.

22. Show that div $kv = k\text{div}v$.

23. Show that $\int_{(0,\pi)}^{\left(3,\frac{\pi}{2}\right)} e^x (\cos y \, dx - \sin y \, dy)$ is path independent.

24. Write the formula for finding the area of a plane region as a line integral over the boundary.

(9 × 2 = 18 marks)

Part C (Short Essay)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Show that the form under integral sign is exact in the plane and evaluate the integral

$$\int_{(-1,-1)}^{(1,1)} e^{-x^2-y^2}(xdx+ydy).$$

26. Solve $2x\tan y\,dx+\sec^2 y\,dy=0$.

27. Find the minimal polynomial $m(t)$ of $A=\begin{pmatrix}2 & 2 & -5\\ 3 & 7 & -15\\ 1 & 2 & -4\end{pmatrix}$.

28. Let $A=\begin{pmatrix}3 & -4\\ 2 & -6\end{pmatrix}$. Find all eigen values and corresponding eigen vectors. Find matrices P and D such that P is non-singular and $D=P^{-1}AP$ is diagonal.

29. Let L be the linear transformation on R^2 that reflects each point P across the line $y=kx$, where $k>0$.

    (a) Show that $v_1=(k,1)$ and $v_2=(1,-k)$ are eigenvectors of L.

    (b) Show that L is diagonalizable, and find a diagonal representation D.

30. Find the straight line L_1 through the point $P:(1,3)$ in the xy-plane and perpendicular to the straight line $L_2: x-2y+2=0$.

31. Evaluate the double integral $\iint_R y^2\,dxdy$ where R is the region bounded by the unit circle in the first quadrant.

32. Solve $2x\tan y\,dx+\sec^2 y\,dy=0$.

33. Verify Greens theorem in the plane for $F=\left[-y^3, x^3\right]$ and the region is the circle $x^2+y^2=25$.

(6 × 5 = 30 marks)

Turn over

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Test for consistency and solve the following system :

(a)
$$\begin{aligned} x_1 + x_2 - 2x_3 + 4x_4 &= 5 \\ 2x_1 + 2x_2 - 3x_3 + x_4 &= 3 \\ 3x_1 + 3x_2 - 4x_3 - 2x_4 &= 1. \end{aligned}$$

(b)
$$\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y - z &= -4 \\ 3x - 2y - z &= 5. \end{aligned}$$

35. Solve :

(a) $2\sin\left(y^2\right)dx + xy\cos\left(y^2\right)dy = 0,\ y(2) = \sqrt{\frac{\pi}{2}}.$

(b) Find the angle between $x - y = 1$ and $x - 2y = -1$.

36. Evaluate $\iint_S (7xi - zk)\cdot n\,dA$ over the sphere $S: x^2 + y^2 + z^2 = 4$ by

(a) Divergence theorem.

(b) Directly.

(2 × 10 = 20 marks)