D 51761 **(Pages : 3)** **Name.......................................**

Reg. No...................................

THIRD SEMESTER (CBCSS—UG) DEGREE EXAMINATION NOVEMBER 2023

Mathematics

MTS 3C 03—MATHEMATICS—3

(2019—2022 Admissions)

Time : Two Hours Maximum : 60 Marks

Part A

All questions can be attended.
Each question carries 2 marks.
Overall Ceiling is 20.

1. If $r(t) = \cos 2t\, i + \sin t\, j$. Find $r'(0)$.

2. Find the curvature of a circle of radius a.

3. Describe the level surfaces of the function $F(x, y, z) = \frac{(xr + y^2)}{z}$.

4. If $F = (x^2y^3 - z^4)\, i + 4x^5y^2z\, j + y^4\, z^6\, k$, find div $(curl\ F)$.

5. Evaluate $\int xy^2\, dy$ on the quarter-circle C defined by $x = 4\cos t,\ y = 4\sin t,\ 0 \le t \le \frac{\pi}{2}$.

6. Find $\int_C ydx + xdy$ on the curves $y = \sqrt{x}$ between $(0, 0)$ and between $(1, 1)$.

7. Convert $(6, \pi/4, \pi/3)$ in spherical coordinates to rectangular co-ordinates.

8. Find the values of $\ln(-1, -i)$.

9. Prove that $\sinh z = \sinh x \cos y + i \cosh x \sin y$.

Turn over

10. Evaluate $\int (z+3)\, dz$, where C is $x = 2t,\ y = 4t - 1,\ 1 \le t \le 3$.

11. Evaluate $\oint_C z^3 - 1 + 3i\, dz$, where C is the circle $|z| = 1$.

12. State Cauchys Integral Formula.

Part B

All questions can be attended.
Each question carries 5 *marks.*
Overall Ceiling is 30.

13. Find an equation of the tangent plane to the graph of $\frac{1}{2}x^2 + \frac{1}{2}y^2 - z = = 4$ at $(1, -1, 5)$.

14. Find the maximum value of the directional derivative of $F(x, y, z) = xy^2 - 4x^2 y + z^2$ at $(1, -1, 2)$ in the direction of $6i + 2j + 3k$.

15. Find the moment of inertia about the y-axis of the thin homogeneous disk $x^2 + y^2 = r^2$ of mass m. Given $\rho(x, y) = \frac{m}{\pi r^2}$.

16. Find the volume of the solid that is under the hemisphere $z = \sqrt{1 - x^2 - y^2}$ and above the region bounded by the graph of the circle $x^2 + y^2 - y = 0$. $V = \iint_R \sqrt{1 - x^2 - y^2}\, dA$.

17. (a) Verify that the function $u(x, y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane.

    (b) Find the harmonic conjugate function of u.

18. Solve the equation $\cos z = 10$.

19. Evaluate $\oint_C \frac{dz}{z^2 + 1}$ where C is the circle $|z| = 3$.

Part C

Answer any **one** *question.*
The question carries 10 *marks.*

20. Verify Stokes theorem. Assume that the surface S is oriented upward. Given $F = z\,i + x\,j + y\,k$; S that portion of the plane $2x + y + 2z = 6$ in the first octant.

21. Let D be the region bounded by the hemisphere $x^2 + y^2 + (z-1)^2 = 9, 1 \le z \le 4,$ and the plane $z = 1.$ Verify the divergence theorem if $F = xi + yj + (z-1)\,k.$

(1 × 10 = 10 marks)