D 111973 **(Pages : 2)** **Name.....**

Reg. No.....

THIRD SEMESTER (CBCSS–UG) DEGREE EXAMINATION, NOVEMBER 2024

Mathematics

MTS 3C 03—MATHEMATICS—3

(2019—2023 Admissions)

Time : Two Hours Maximum Marks : 60

Part A

All *questions can be answered.*
Each question carries 2 *marks.*
(Ceiling 20 *marks)*

1. If $r(t)=\langle f(t),g(t),h(t)\rangle$, where f, g and h are differentiable, then prove that $r'(t)=\langle f'(t),g'(t),h'(t)\rangle$.
2. The position of a moving particle is given by $r(t)=t^2i+tj+\frac{5}{2}tk$. Find $v(2)$ and $a(2)$.
3. Describe the level surfaces of the function $f(x,y)=y^2-x$.
4. If $\mathrm{F}=\left(x^2y^3-z^4\right)i+4x^5y^2zj+y^4z^6k$, find curl F.
5. Evaluate $\int xy^2ds$ on the quarter-circle C defined by $x = 4\cos t$, $y=4\sin t, 0\le t\le\frac{\pi}{2}$.
6. Find $\int_C ydx+xdy$ on the curve $y = x^3$ between (0, 0) and between (1, 1).
7. Convert $\left(8,\frac{\pi}{3},7\right)$ in cylindrical co-ordinates to rectangular co-ordinates.
8. Find the values of ln(– 2).
9. Prove that $\cosh^2 z+\sinh^2 z=1$.
10. Evaluate $\int\bar{z}dz$, where C is given by $x=3t, y=t^2, -1\le t\le 4$.
11. Evaluate $\oint_C e^z dz$, where C is the circle $|z| = 2$.
12. Derive Cauchys inequality.

Turn over

Part B

All *questions can be answered.*
Each question carries 5 *marks.*
(Ceiling 30 *marks)*

13. Find the directional derivative $f(x, y) = 2x^2y^3 - 6xy$ at (1, 1) in the direction of a unit vector whose angle with the positive x-axis is $\frac{\pi}{6}$.

14. Find an equation of the tangent plane to the graph of $x^2 - 4y^2 + z^2 = 16$ at (2, 1, 4).

15. Evaluate the double integral $\iint_R e^{x+3y} dA$ over the region bounded by the graphs of $y = 1$, $y = 2$, $y = x$ and $y = -x + 5$.

16. Evaluate $\oint_C \left(x^5 + 3y\right) dx + \left(2x - e^{y^3}\right) dy$, where C is the circle $(x-1)^2 + (y-5)^2 = 4$.

17. Find the volume of the solid in the first Octant bounded by the graphs of $z = 1 = y^2$, $y = 2x$ and $x = 3$.

18. Solve the equation $\cos z = 10$.

19. Find an upper bound for the absolute value of $\oint_C \frac{e^z}{z+1} dz$, where C is the circle $|z| = 4$.

Section C

Answer any **one** *questions.*
The question carries 10 *marks.*

20. Let S be the part of the cylinder $z = 1 - x^2$ for $0 \le x \le 1, -2 \le y \le 2$. Verify Stoke theorem for the vector field $F = xy\,i + yz\,j + xz\,k$. Assume S is oriented upward.

21. Find the moment of inertia about the z-axis of the homogeneous solid bounded between the spheres $x^2 + y^2 + z^2 = a^2$ and $x^2 + y^2 + z^2 = b^2, a < b$.

(1 × 10 = 10 marks)