

FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION, APRIL 2021

Mathematics

MTS 4C 04—MATHEMATICS—4

Time : Two Hours

Maximum : 60 Marks

Section A

*Answer at least eight questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 24.*

1. If $x = C_1 \cos 4t + C_2 \sin 4t$ is a solution of $x'' + 16x = 0$, $x\left(\frac{\pi}{2}\right) = -2$, $x'\left(\frac{\pi}{2}\right) = 1$, then find C_1 and C_2 .
2. Write any two solutions of $y'' = y'$ by inspection.
3. Solve $\frac{dy}{dx} = \sin 5x$.
4. Check whether $\frac{dy}{dx} = \frac{xy^2 - \cos x \sin x}{y(1-x^2)}$ is an exact differential equation.
5. Write the integrating factor corresponding to the differential equation $\frac{xdy}{dx} - 4y = x^6 e^x$.
6. Show that $y = 3e^{2x} + e^{-2x} - 3x$ is a unique solution of $y'' - 4y = 12x$, $y(0) = 4$, $y'(0) = 1$.
7. Check whether $y_1 = e^{3x}$ and $y_2 = e^{-3x}$ are linearly independent solutions of $y'' - 9y = 0$.
8. If $y_1 = x^2$ is a solution of $x^2 y'' - 3xy' + 4y = 0$, find a second solution.
9. Find the Laplace transform of e^{-3t} .
10. Find the inverse Laplace transform of $\frac{6-2s}{s^2+4}$.
11. Check whether $f_1(x) = x$, $f_2(x) = \cos 2x$ are orthogonal in $[-\pi/2, \pi/2]$.
12. Show that the partial differential equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ is elliptic.

(8 × 3 = 24 marks)

Turn over

Section B

*Answer at least five questions.
Each question carries 5 marks.
All questions can be attended.
Overall Ceiling 25.*

13. Solve $(x^2 + y^2)dx + (x^2 - xy)dy = 0$.
14. Solve the initial value problem $\frac{dy}{dx} = (-2x + y)^2 - 7, y(0) = 0$.
15. Find the particular integral of $(D - 3)^3 y = xe^{-2x}$.
16. Using the method of variation of parameters solve $y'' + 3y' + 2y = 3e^{-2x} + x$.
17. Convert the equation $xy'' - 3y' + x^{-1}y = x^2$ as a linear equation with constant co-efficients.
18. Find the Laplace transform of $\frac{\cos at - \cos bt}{t}$.
19. Find the half range sine series of $f(x) = \cos x$ in $0 < x < \pi$.

(5 × 5 = 25 marks)

Section C

*Answer any one question.
Each question carries 11 marks.*

20. Solve the differential equation $\frac{d^2y}{dt^2} - 3\frac{dy}{dt} + 2y = 4, y(0) = 2, y'(0) = 3$, using Laplace Transform.
21. Obtain the Fourier series of $f(x) = x^2 - 2$ in the interval $(-2, 2)$.

(1 × 11 = 11 marks)