D 120575 **(Pages : 3)** **Name.......................................**

Reg. No...................................

FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2025

Mathematics

MTS 4C 04—MATHEMATICS—4

(2019—2023 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Ceiling is 20.

1. Solve $(1+x)\,dy - ydx = 0$.

2. Solve initial value problem $\frac{dy}{dx} + y = x,\ y(0) = 4$.

3. Solve $\left(e^{2x} - y\cos xy\right)dx + \left(2xe^{2y} - x\cos xy + 2y\right)dy = 0$.

4. The function $y_1 = x^2$ is a solution of $x^2y'' - 3xy' + 4y = 0$. Find the general solution on the interval $(0, \infty)$.

5. Solve $y'' - 10y' + 25y = 0$.

6. Find a particular solution of $y'' - 5y' + 4y = 8e^x$.

7. Let $f(t) = 4t^2 - 5\sin 3t$. Find $\mathscr{L}(f(t))$.

8. Evaluate $\mathscr{L}(f(t))$ for $f(t) = \begin{cases} 0, & 0 \le t < 3 \\ 2, & t \ge 3 \end{cases}$.

Turn over

9. Evaluate $\mathcal{L}^{-1}\left(\dfrac{-2s+6}{s^2+4}\right)$.

10. Show that the functions $f_1(x) = x^3, f_2(x) = x^2 + 1$ are orthogonal on $[-2, 2]$.

11. Show that the partial differential equation $\dfrac{\partial^2 u}{\partial x^2} = 9\dfrac{\partial^2 u}{\partial x \partial y}$ is hyperbolic.

12. State superposition principle.

Section B

Answer any number of questions.
Each question carries 5 marks.
Ceiling is 30.

13. Solve $xydx + \left(2x^2 + 3y^2 - 20\right)dy = 0$ by finding an appropriate integrating factor.

14. Solve the initial value problem $\dfrac{dy}{dx} = (-2x + y)^2 + 7, y(0) = 0$.

15. Solve the initial value problem $y'' + y = 4x + 10\sin x, y(\pi) = 0, y'(\pi) = 2$.

16. Solve $x^2 y'' - 3xy' + 3y = 2x^4 e^x$.

17. Use the Laplace transform to solve the initial value problem $\dfrac{dy}{dt} + 3y = 13\sin 2t, y(0) = 6$.

18. Evaluate :

    a) $e^t * \sin t$.

    b) $\mathcal{L}\left(e^t * \sin t\right)$.

19. Show that the set $\{1, \cos x, \cos 2x, ...\}$ is orthogonal on the interval $[-\pi, \pi]$. Also find norm of each functions in the set.

Section C

Answer any **one** *question.*
The question carries 10 *marks.*

20. Solve $4y'' + 36y = \csc 3x$ by variation of parameters.

21. Find the Fourier series of $f(x) = \begin{cases} 0, & -\pi < x < 0 \\ x^2, & 0 \le x < \pi \end{cases}$ on the interval $[-\pi, \pi]$.

(1 × 10 = 10 marks)